Reinstating Floyd-Steinberg: Improved Metrics for Quality Assessment of Error Diffusion Algorithms

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Introduction

- Halftoning is the process of converting a continuous tone image into another image with a reduced number of tones.
Ordered dithering

- Tiles a dither matrix (e.g. Bayer, void-and-cluster) for use as threshold values.
- Average quality, fast, parallelisable.
Error diffusion

- Uses feedback through an error diffusion matrix (eg. Floyd-Steinberg, Stucki, JaJuNi...).
- Good quality, hardly parallelisable.
Direct binary search

- Uses a human visual system model to simulate eye behaviour and minimise a global distance.
- Excellent quality, yet very slow (iterative).
What is “quality”? 

- One possible metric:

\[ E(h, b) = \frac{(||v * h_{i,j} - v * b_{i,j}||_2)^2}{wh} \]

- \( h \): original image
- \( b \): dithered image
- \( v \): human visual system filter

- Smaller error = higher quality
The human visual system model
Relative quality of algorithms

- Using *lena.tga* and a Gaussian HVS:
  - 8×8 Bayer dithering: $E = 0.06352$
  - Floyd-Steinberg dithering: $E = 0.01847$
  - Direct binary search: $E = 0.00984$

- Other images show a similar trend
Image displacement

- Floyd-Steinberg kernel:
  \[
  \frac{1}{16} \quad \begin{array}{ccc}
  - & x & 7 \\
  3 & 5 & 1
  \end{array}
  \]

- Error is propagated to the right and bottom, which shows in the image:
Image displacement

- Slight formula modification:

\[
E_{dx,dy}(h, b) = \frac{(\|v \ast h_{i,j} - v \ast t_{dx,dy} \ast b_{i,j}\|_2)^2}{wh}
\]

- \( t_{dx,dy} \) translates the image by \((dx,dy)\)

- For most error-diffused images, \( E_{dx,dy} \) has a local minimum \( E_{min} \) which is not \((0,0)\)

- \( E_{min} \) is our new quality metric
$E_{dx,dy}$ variations for *lena.tga*

- F-S displacement for *lena.tga* is (0.28, 0.22):
(dx,dy)_{min} variations

- Histogram on a sample of 10,000 images:

- Floyd-Steinberg displacement is (0.16, 0.28)

- We introduce $E_{\text{fast}} = E_{0.16, 0.28}$ which is statistically a better metric than $E$. 
Displacement for other algorithms

- Jarvis-Judice-Ninke: (0.26, 0.76)
- Ostromoukhov: (0.0, 0.19)
- Serpentine optimum: (0.0, 0.34)
Application: improving Floyd-Steinberg

- The idea: find the best Floyd-Steinberg-like diffusion kernel by computing $E_{min}$ for many sample images and diffusion kernels.
- We use CPUShare (www.cpushare.com) for cheap and secure distributed computing.
Results

• Result #1: best kernel is \{6, 3, 5, 2\}, original
  Floyd-Steinberg \{7, 3, 5, 1\} is 2\textsuperscript{nd} best

• Result #2: using $E$ instead of $E_{\text{min}}$ elects poor kernels such as \{7, 3, 6, 0\}

• Result #3: \{7, 4, 5, 0\} is a much better kernel for serpentine scan than the original Floyd-Steinberg
Future work

- Find a link between \((dx, dy)_{min}\) and the ED kernel
- Optimise other widely-used ED kernels
- Experiment with more complex HVS models